MATH 54 - HINTS TO HOMEWORK 2

PEYAM TABRIZIAN

Here are a couple of hints to Homework 2! Enjoy! :)

SECTION 1.3: VECTOR EQUATIONS

1.3.7. Here's a cool trick! **Any** vector in $\mathbb{R}^2$ is a linear combination of two linearly independent vectors! So the answer is immediately $\boxed{\text{yes}}$

1.3.22. In other words, find an inconsistent system of 3 equations! Think about why this is true!

1.3.25. Careful! A set is not the same as the span of a set. In particular, $\mathbf{b}$ is not in $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ because it is not equal to either of those vectors. However, it might be in the span of those 3 vectors! Also, for (c), remember that $\mathbf{a_1}$ is always in the span of $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$.

SECTION 1.4: THE MATRIX EQUATION $Ax = b$

1.4.17, 1.4.18. Row-reduce! Also, use Theorem $4(d)$ on page 45.

1.4.29. The easiest way to do this is find a matrix in row-echelon form that has this property, and then just interchange two rows! For example, the following matrix works:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.4.34. Ignore the word 'unique'. Then Theorem $4(a)$ holds, and in particular Theorem $4(c)$ holds, which solved the problem!

SECTION 1.5: SOLUTION SETS OF LINEAR SYSTEMS

1.5.14. The line that goes through $\begin{bmatrix} 0 \\ 8 \\ 2 \\ 0 \end{bmatrix}$ and with 'slope' $\begin{bmatrix} 3 \\ 1 \\ -5 \\ 1 \end{bmatrix}$

Date: Monday, September 5th, 2011.

1.5.24.

(a) **F** (nontrivial means not all entries are 0. For a counterexample, let A be the zero matrix!)
(b) **T**
(c) **F** (no, it means $\mathbf{b} = \mathbf{0}$)
(d) **F** (no, draw a picture, or see page 55)
(e) **T** (see page 55)

1.5.29, 1.5.30, 1.5.31, 1.5.32. Nontrivial means $\mathbf{x} \neq \mathbf{0}$. The best way to do this is to draw a picture of what the reduced-echelon form of the matrix looks like! Also, for (b), if one of the rows of A is a row of zeros, then the equation $A\mathbf{x} = \mathbf{b}$ has no solution!

Section 1.6: Applications of linear systems

Ignore this section if you want, it's more for your personal entertainment! :)

Section 1.7: Linear independence

1.7.9. Notice that $\mathbf{v_2} = -3\mathbf{v_1}$, so in other words, for what values of h is $\mathbf{v_3}$ a multiple of $\mathbf{v_1}$. For (b), the set is always linearly dependent because $\mathbf{v_2} = -3\mathbf{v_1}$ already.

1.7.11, 1.7.13. Row-reduce!

1.7.15, 1.7.17. A set with more than 2 elements in $\mathbb{R}^2$ is always linearly dependent. A set with the zero-vector is always linearly dependent.

1.7.21.

(a) **F** (the equation $A\mathbf{x} = \mathbf{0}$ **always** has the trivial solution, no matter what the columns of A look like!)
(b) **F** (for example, $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \right\}$ doesn't satisfy this! The correct statement should be: there is **some** vector such that $\cdots$)
(c) **T** (in other words, 5 vectors in $\mathbb{R}^4$ are linearly dependent)
(d) **T** (otherwise the set would be linearly independent)

1.7.22.

(a) **T** (end of page 69)
(b) **F** (for example, $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ is linearly dependent)
(c) **T** ($\mathbf{z}$ is a linear combination of $\mathbf{x}$ and $\mathbf{y}$!)
(d) **F** (for example, $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ is linearly dependent)

1.7.33, 1.7.34. Remember that a set is linearly dependent if there's a relationship between the vectors in the set. Also, a set with the zero vector is always linearly dependent.

1.7.35. False ($\mathbf{v_1}$ could be the zero vector!)

1.7.36. False (choose $\mathbf{v_1} = \mathbf{v_2} = \mathbf{v_4}$ and $\mathbf{v_3}$ linearly independent from $\mathbf{v_1}$! The point is for linear independence, you have to consider the set as a whole!)

SECTION 1.8: INTRODUCTION TO LINEAR TRANSFORMATIONS

1.8.3, 1.8.9, 1.8.11. Just solve the equation $A\mathbf{x} = \mathbf{b}$, where in $1.8.9$, $\mathbf{b}$ is the zero vector!

SECTION 1.9: THE MATRIX OF A LINEAR TRANSFORMATION

For **all** of those questions, all you need to find is $T(\mathbf{e_1}), T(\mathbf{e_2}), \cdots$ and group the terms in a matrix!

1.9.23.

(a) **T** (in other words, if you know $T(\mathbf{e_1}), T(\mathbf{e_2}), \cdots, T(\mathbf{e_n})$, you know T)
(b) **T** (see example 3)
(c) **F** (the composition of two linear transformations is a linear transformation, see chapter 4)
(d) **F** (onto means every vector in $\mathbb{R}^n$ is in the image of T)
(e) **F** (let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, then the columns of A are linearly independent, and hence T is one-to-one by theorem $12b$)

1.9.24.

(a) **F**
(b) **T**
(c) **T**
(d) **T**
(e) **T**